

# RADIATION AND MHD FLOW A SEMIFINITE MOVING VERTICAL PLATE WITH VISCOUS DISSIPATION

Amit Saraswat Rajesh Kumar and R.K.shrivastava

## Abstract:

The objectives of the present study are to investigate the radiation effects on unsteady magneto hydrodynamic flow of a chemically reacting fluid past a semi-infinite vertical plate with viscous dissipation. The method of solution can be applied for small perturbation approximation. Numerical results for the velocity, the temperature and the concentration are shown graphically. The expressions for the skin-friction, Nusselt number and Sherwood number are obtained.

**Keyword:** Heat and mass transfer, magneto hydrodynamic flow, viscous dissipation, Radiation, Chemical reaction, semi-infinite vertical plate, Numerical results .

## INTRODUCTION:

For some industrial application such as glass production and furnace design in space technology applications, cosmoial flight aerodynamics rocket, propulsion systems, plasma physics which operate at higher temperatures, radiation effects can be significant. Soundalgekar and Takhar (1993) considered the radiative free convection flow of an optically thin grey-gas past a semi-infinite vertical plate. Radiation effects on mixed convection along an isothermal vertical plate were studied by Hussain and Takhar (1996). Raptis and Perdakis (1999) have studied the effects of thermal radiation and free convection flow past a moving vertical plate.

Chamkha et al. (2001) analyzed the effects of radiation on free convection flow past a semi-infinite vertical plate with mass transfer. Muthucumaraswamy and Ganesan (2003) have studied the radiation effects on flow past an impulsively started infinite vertical plate with variable temperature. Prakash and Ogulu (2006) have investigated an unsteady two-dimensional flow of a radiating and chemically reacting fluid with time dependent suction.

In many chemical engineering processes, there does occur the chemical reaction between a foreign mass and the fluid in which the plate is moving. These processes take place in numerous industrial applications viz., Polymer production, manufacturing of ceramics or glass ware and food procession. Das et al. (1994) have studied the effects of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction. Muthucumaraswamy (2002) has studied the effects of reaction on a moving isothermal vertical infinitely long surface with suction. Recently, Reddy et al. (2008) have

studied on radiation and mass transfer flow past a semi-infinite moving vertical plate with viscous dissipation.

The objective of the present paper is to analyze the radiation and magneto hydrodynamic effects on an unsteady two-dimensional laminar convective boundary layer flow of a viscous, incompressible, chemically reacting fluid a semi-infinite vertical plate with suction, by taking into account the effects of viscous dissipation. The equations of continuity, linear momentum, energy and diffusion, which govern the flow field, are solved by using a regular perturbation method. The behavior of the velocity, temperature, concentration has been discussed for variations in the governing parameters.

## FORMULATION OF THE PROBLEM:

An unsteady two-dimensional laminar boundary layer flow of a viscous, incompressible, radiating fluid along a semi-infinite vertical plate in the presence of a uniform magnetic field and concentration buoyancy effects is considered, by taking the effect of viscous dissipation into account. The  $x'$ -axis is taken along the vertical infinite plate in the upward direction and the  $y'$ -axis normal to the plate. The level of concentration of foreign mass is assumed to be low, so that the Soret and Dufour effects are negligible. Now, under the usual Boussinesq's approximation, the flow field is governed by the following equations:

$$\frac{\partial v'}{\partial y'} = 0 \quad \dots(2.1)$$

$$\frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) - \frac{\sigma B_0^2}{\rho} u' \quad (2.2)$$

$$\frac{\partial T}{\partial t'} + v' \frac{\partial T}{\partial y'} = \alpha \frac{\partial^2 T}{\partial y'^2} - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial y'} + \frac{\mu}{\rho C_p} \left( \frac{\partial u'}{\partial y'} \right)^2 \quad (2.3)$$

$$\frac{\partial C}{\partial t'} + v' \frac{\partial C}{\partial y'} = D \frac{\partial^2 C}{\partial y'^2} - K_r'^2 (C - C_\infty) \quad \dots(2.4)$$

Where  $u', v'$  are the velocity components in  $x', y'$  dimensions respectively,  $t'$  - time,  $\rho$  - the fluid density,  $\nu$  - the kinetic viscosity,  $C_p$  - the specific heat at constant pressure,  $g$  - the acceleration due to gravity,  $\beta$  and  $\beta^*$  - the volumetric coefficient of thermal and concentration

expansion,  $T$  - the dimensional temperature,  $C$  - the dimensional concentration,  $\alpha$  - the fluid thermal diffusivity,  $\mu$  - coefficient of viscosity,  $D$  - the mass diffusivity  $K_r r_c$  - the chemical reaction parameter and  $B_0^2$  -magnetic field parameter.

The boundary conditions for the velocity, temperature and concentration fields are:

$$\left. \begin{aligned} u' = U_0, T = T_w + \varepsilon(T_w - T_\infty)e^{nt}, C = C_w + \varepsilon(C_w - C_\infty)e^{nt} \text{ at } y' = 0 \\ u' \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty \text{ as } y' \rightarrow \infty \end{aligned} \right\} \quad (2.5)$$

Where  $U_0$  is the scale of free stream velocity,  $C_w$  and  $T_w$  are the wall dimensional concentration and temperature respectively,  $C_\infty$  and  $T_\infty$  are the free stream dimensional concentration and temperature, respectively,  $n'$  the constant.

By using Rosseland approximation (Brewster (1992)), the radiative heat flux is given by

$$q_r = -\frac{4\sigma_s}{3K_e} \frac{\partial T^4}{\partial y'} \quad \dots(2.6)$$

Where  $\sigma_s$  is the Stefan-Boltzmann constant and  $K_e$  - the mean absorption coefficient. It should be noted that by using the Rosseland approximation, the present analysis is limited to optically thick fluids. If temperature differences within the flow are sufficient, small, then "(2.6)" can be linearised by expanding  $T^4$  in to the Taylor series about  $T_\infty$ , which after neglecting higher order terms take the form

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4 \quad \dots(2.7)$$

In view of "(2.6)" and "(2.7)", "(2.3)" reduces to

$$\frac{\partial T}{\partial t'} + v' \frac{\partial T}{\partial y'} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y'^2} + \frac{16\sigma_s}{3\rho C_p K_e} T_\infty^3 \frac{\partial^2 T}{\partial y'^2} + \frac{\mu}{\rho C_p} \left( \frac{\partial u'}{\partial y'} \right)^2 \quad (2.8)$$

From the continuity "(2.1)", it is clear that the suction velocity normal to the plate is either a constant or a function of time. Hence, it is assumed in the form

$$v' = -V_0 (1 + \varepsilon A e^{nt'}) \quad \dots(2.9)$$

Where  $A$  is a real positive constant,  $\varepsilon$  and  $\varepsilon A$  are small less than unity, and  $V_0$  is a non-zero positive constant, the negative sign indicates that the suction is towards the plate.

In order to write the governing equations and boundary conditions in dimensional less, the following non-dimensional quantities introduced.

$$\left. \begin{aligned} u = \frac{u'}{U_0}, \quad y = \frac{V_0 y'}{\nu}, \quad t = \frac{V_0^2 t'}{\nu}, \quad Pr = \frac{\rho C_p \nu}{k} = \frac{\nu}{\alpha} \\ \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi = \frac{C - C_\infty}{C_w - C_\infty}, \quad n = \frac{n' \nu}{V_0^2}, \quad Sc = \frac{\nu}{D} \\ R = \frac{16\sigma_s T_\infty^3}{3K_e k}, \quad M = \frac{\sigma B_0^2 \nu}{\rho V_0^2}, \quad Ec = \frac{U_0^2}{C_p (T_w - T_\infty)}, \\ K_r^2 = \frac{K_r^2 \nu}{V_0^2}, \quad Gr = \frac{g \beta \nu (T_w - T_\infty)}{U_0 V_0^2}, \quad Gc = \frac{g \beta^* \nu (C_w - C_\infty)}{U_0 V_0^2} \end{aligned} \right\} \quad (2.10)$$

In view of the (2.9) and (2.10),(2.2), (2.8), and(2.4) reduce to the following dimensions form.

$$\frac{\partial u}{\partial t} - (1 + \varepsilon A e^{nt}) \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + Gr \theta + Gc \phi - Mu \quad (2.11)$$

$$\frac{\partial \theta}{\partial t} - (1 + \varepsilon A e^{nt}) \frac{\partial \theta}{\partial y} = \left( \frac{1 + R}{Pr} \right) \frac{\partial^2 \theta}{\partial y^2} + Ec \left( \frac{\partial u}{\partial y} \right)^2 \quad (2.12)$$

$$\frac{\partial \phi}{\partial t} - (1 + \varepsilon A e^{nt}) \frac{\partial \phi}{\partial y} = \frac{1}{Sc} \frac{\partial^2 \phi}{\partial y^2} - K_r^2 \phi \quad (2.13)$$

Where  $Gr$ ,  $Gc$ ,  $Pr$ ,  $R$ ,  $Ec$ ,  $Sc$ ,  $M$  and  $K_r$  are the thermal Grashof number, Solutal Grashof number, Prandtl number, Radiation parameter, Eckert number, Schmidt number, Magnetic field parameter and chemical reaction parameter respectively.

The corresponding dimensions boundary conditions are

$$\left. \begin{aligned} u = 1, \quad \theta = 1 + \varepsilon e^{nt}, \quad \phi = 1 + \varepsilon e^{nt} \text{ at } y = 0 \\ u \rightarrow 0, \quad \theta \rightarrow 0, \quad \phi \rightarrow 0 \text{ as } y \rightarrow \infty \end{aligned} \right\} \quad (2.14)$$

### SOLUTIONS OF THE PROBLEM:

Equations (2.11) - (2.13) are coupled, non-linear partial differential equations and these cannot be solved in closed form. However, these equations can be reduced to a set of ordinary differential equations, which can be solved analytically. This can be done by representing the velocity, temperature and concentration of the fluid in the neighborhood of the plate as

$$\left. \begin{aligned} u(y,t) = u_0(y) + \varepsilon e^{nt} u_1(y) + O(\varepsilon)^2 + \dots \\ \phi(y,t) = \phi_0(y) + \varepsilon e^{nt} \phi_1(y) + O(\varepsilon)^2 + \dots \\ \theta(y,t) = \theta_0(y) + \varepsilon e^{nt} \theta_1(y) + O(\varepsilon)^2 + \dots \end{aligned} \right\} \quad (3.1)$$

Substituting (3.1) in equations (2.11) - (2.13) and equating the harmonic and non-harmonic terms, and neglecting the higher - order terms of  $O(\varepsilon^2)$ , we obtain

$$u''_0 + u'_0 - Mu_0 = -Gr\theta_0 - Gc\phi_0 \quad \dots(3.2)$$

$$u''_1 + u'_1 - (n + M)u_1 = -Au'_0 - Gr\theta_1 - Gc\phi_1 \quad \dots(3.3)$$

$$\theta''_0 + h\theta'_0 = -hEc u'^2_0 \quad \dots(3.4)$$

$$\theta''_1 + h\theta'_1 - hn\theta_1 = -hEc\theta'_0 - 2u'_0 u'_1 hEc \quad \dots(3.5)$$

$$\phi''_0 + Sc\phi'_0 - ScK_r^2 \phi_0 = 0 \quad \dots(3.6)$$

$$\phi''_1 + Sc\phi'_1 - Sc(n + K_r^2)\phi_1 = -ASc\phi'_0 \quad \dots(3.7)$$

Where  $h = \frac{Pr}{1 + R}$  and prime denotes ordinary differentiation with respect to  $y$ .

The corresponding boundary conditions can be written as

$$\left. \begin{aligned} u_0 = 1, u_1 = 0, \theta_0 = 1, \theta_1 = 0, \phi_0 = 1, \phi_1 = 0 \quad \text{at } y=0 \\ u_0 \rightarrow 0, u_1 \rightarrow 0, \theta_0 \rightarrow 0, \theta_1 \rightarrow 0, \phi_0 \rightarrow 0, \phi_1 \rightarrow 0 \text{ as } y \rightarrow \infty \end{aligned} \right\} \dots(3.8)$$

The equations (3.2) - (3.7) are still coupled non-linear, whose exact solutions are not possible. So, we can expand  $u_0, u_1, \theta_0, \theta_1, \phi_0, \phi_1$  in terms of  $Ec$  (Eckert number) in the following form as the Eckert number is very small for incompressible flows.

$$\left. \begin{aligned} u_0(y) &= u_{01}(y) + Ec u_{02}(y) \\ u_1(y) &= u_{11}(y) + Ec u_{12}(y) \\ \theta_0(y) &= \theta_{01}(y) + Ec \theta_{02}(y) \\ \theta_1(y) &= \theta_{11}(y) + Ec \theta_{12}(y) \\ \phi_0(y) &= \phi_{01}(y) + Ec \phi_{02}(y) \\ \phi_1(y) &= \phi_{11}(y) + Ec \phi_{12}(y) \end{aligned} \right\} \dots(3.9)$$

Substituting (3.9) in equations (3.2) - (3.7), equating the coefficients of  $Ec$  to zero neglecting the terms in  $Ec^2$  and higher order, and solving all the ordinary differential equations. We obtain the velocity, temperature and concentration distribution in the boundary layer as:

$$\begin{aligned} \theta(y,t) = & e^{-hy} + Ech(L_{16}e^{-m_6y} - L_{10}e^{-2m_6y} - L_{11}e^{-2m_6y} - L_{12}e^{-2m_6y} - L_{13}e^{-m_6y} + L_{14}e^{-m_6y} \\ & + L_{15}e^{-m_6y}) + \varepsilon e^{nt} [L_{26}(e^{-m_7y} - e^{-m_7y}) + hEc\{L_{42}e^{-m_7y} - hA(L_{44}e^{-m_7y} + \\ & L_{45}e^{-2m_7y} + L_{46}e^{-2m_7y} + L_{47}e^{-2m_7y} + L_{48}e^{-m_7y} - L_{49}e^{-m_7y} - L_{50}e^{-m_7y}) \\ & - 2(L_{51}e^{-m_{10}y} + L_{52}e^{-2m_{10}y} - L_{53}e^{-m_{10}y} - L_{54}e^{-m_{10}y} - L_{55}e^{-m_{11}y} - L_{56}e^{-m_{12}y} \\ & - L_{57}e^{-m_{13}y} + L_{58}e^{-2m_{13}y} + L_{59}e^{-m_{14}y})\}] \end{aligned} \quad \dots(3.11)$$

$$\phi(y,t) = e^{-m_2y} + \varepsilon e^{nt} (L_{28}e^{-m_8y} + L_{27}e^{-m_2y}) \quad (3.12)$$

Where the expressions for the constants are given in the Appendix.

Knowing the velocity, the temperature and the concentration fields, the skin-friction, Nusselt number and the Sherwood number at the plate can be obtained, which in the non-dimensional forms are given by

$$\begin{aligned} C_f = \frac{\tau'_w}{\rho U_0 V_0} &= \left( \frac{\partial u}{\partial y} \right)_{y=0} = \left( \frac{\partial u_0}{\partial y} + \varepsilon e^{nt} \frac{\partial u_1}{\partial y} \right)_{y=0} \\ &= L_{107} + EcGrh(L_{108} + L_{109}) + \varepsilon e^{nt} \{L_{110} + L_{111} + Ec(L_{112} + L_{113})\} \quad \dots(3.13) \end{aligned}$$

$$\begin{aligned} Nu = -x \frac{\left( \frac{\partial T}{\partial y} \right)'}{(T_w - T_\infty)} &\Rightarrow Nu Re_x^{-1} = - \left( \frac{\partial \theta}{\partial y} \right)_{y=0} = - \left[ \frac{\partial \theta_0}{\partial y} + \varepsilon e^{nt} \frac{\partial \theta_1}{\partial y} \right]_{y=0} \\ &= -h + Ech(L_{114} + L_{115}) + \varepsilon e^{nt} [L_{116} + Ech\{L_{117} - hA(L_{118} + L_{119}) \\ & - 2(L_{120} + L_{121})\}] \end{aligned} \quad \dots(3.14)$$

Where  $Re_x = \frac{V_{0x}}{\nu}$  is the local Reynolds number.

$$\begin{aligned}
 u(y,t) = & (L_3 e^{-m_3 y} - L_1 e^{-ly} - L_2 e^{-m_2 y}) + EcGrh(L_{25} e^{-m_{25} y} - L_{17} e^{-m_{17} y} + L_{18} e^{-2m_4 y} \\
 & + L_{19} e^{-2m_5 y} + L_{20} e^{-2m_6 y} + L_{21} e^{-m_4 y} - L_{22} e^{-m_5 y} - L_{23} e^{-m_6 y}) + \varepsilon e^{nt} ((L_{34} e^{-m_6 y} \\
 & + L_{29} e^{-m_3 y} - L_{30} e^{-m_4 y} - L_{31} e^{-m_5 y} - L_{32} e^{-m_6 y} - L_{33} e^{-m_3 y}) + Ec(L_{94} e^{-m_4 y} + \\
 & L_{95} e^{-m_5 y} + L_{96} e^{-2m_4 y} + L_{97} e^{-2m_5 y} + L_{98} e^{-2m_6 y} + L_{99} e^{-m_4 y} + L_{100} e^{-m_5 y} + L_{101} e^{-m_6 y} \\
 & - L_{102} e^{-m_3 y} + L_{103} e^{-m_4 y} + L_{104} e^{-m_{10} y} + L_{105} e^{-m_{11} y} - L_{106} e^{-m_{12} y})) \dots(3.10)
 \end{aligned}$$

$$\begin{aligned}
 Sh = -x \frac{\left(\frac{\partial C}{\partial y'}\right)}{(C_w - C_\infty)} \Rightarrow Sh Re_x^{-1} = -\left(\frac{\partial \phi}{\partial y}\right)_{y=0} = -\left[\frac{\partial \phi_0}{\partial y} + \varepsilon e^{nt} \frac{\partial \phi_1}{\partial y}\right]_{y=0} \\
 = -m_2 + \varepsilon e^{nt} (-m_8 L_{28} - m_2 L_{27}) \dots(3.15)
 \end{aligned}$$

**RESULTS AND DISCUSSION:**

In the preceding sections, the problem of an unsteady MHD free convective flow of a viscous, incompressible, radiating and dissipating fluid through porous medium past a semi-finite plate with chemically reacting was formulated and solved by means of a perturbation method. The expression for the velocity, temperature and concentration were obtained. To illustrate the behavior of these physical quantities, numeric values were computed with respect to the variations in the governing parameters viz., the thermal Grashof number Gr, the solutal Grashof number Gc, Magnetic field parameter M, Prandtl number Pr (Pr = 0.71), Schemidt number Sc, the radiation parameter R, Eckert number Ec and chemical reaction parameter Kr.

The velocity profiles for different values of the thermal Grashof number Gr are described in Figure - (7.1), it is observed that an increase in Gr, leads to arise in the values of velocity. Hence the positive value of Gr corresponds to cooling of the plate. In addition, it is observed that the velocity increases rapidly near the wall of the plate as Grashof number increases and then decays to the free stream velocity. For the case of different values of Solutal Grashof number Gc, the velocity profiles in the boundary layer as Shown in Figure - (7.2). It is observed that an increase in Gc, leads to a rise in the values of velocity.

The velocity profiles of boundary layer flow are plotted in Figure -(7.3) for different values of magnetic field parameter M, it is observed that an increases in the magnetic field parameter M results decrease in the velocity.

Figures - (7.4) and (7.5) shows that the velocity and temperature profiles for different values of the thermal radiation parameter R. It is noticed that an increase in the thermal radiation parameter results increase in the velocity

and temperature with in boundary layer, as well as increased the thickness of the velocity and temperature boundary layers.

The velocity and temperature profiles of boundary layer flow for different values of Prandtl number Pr are described in figures - (7.6) and (7.7), it is shown that the velocity and temperature rapidly decreases with increasing the value of Prandtl number Pr.

The effects of the viscous dissipation parameter i.e., Eckert number on the velocity and temperature are shown in figures - (7.8) and (7.9). Greater viscous dissipative heat causes a rise in the temperature as well as the velocity.

The effects of Schmidt number on the velocity and concentration are shown in figures - (7.10) and (7.11). As the Schmidt number increases, the concentration decreases. This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity. Reductions in the velocity and concentration distributions are accompanied by simultaneous reductions in the velocity and concentration boundary layers.

Figures - (7.12) and (7.13) illustrates the behavior velocity and concentration for the different values of chemical reaction parameter Kr. It is observed that an increase in chemical reaction parameter Kr leads to a decrease in both the values of velocity and concentration.

**CONCLUSIONS:**

The theoretical solution for the effects of the radiation effects on unsteady magneto hydrodynamic flow of a chemically reacting fluid past a semi-infinite vertical plate with viscous dissipation. The method of solution can be applied for small perturbation approximation. Numerical results for the velocity, the temperature and the concentration are shown graphically. The study concludes the following results.

An increase in Gr, leads to arise in the values of velocity. The velocity increases rapidly near the wall of the plate as Grashof number increases and then decays to the free stream velocity.

An increase in Gc, leads to a rise in the values of velocity.

An increase in the magnetic field parameter M results decrease in the velocity.

An increase in the thermal radiation parameter results increase in the velocity and temperature with in boundary layer, as well as increased the thickness of the velocity and temperature boundary layers.

The velocity and temperature rapidly decreases with increasing the value of Prandtl number Pr.

Greater viscous dissipative heat causes a rise in the temperature as well as the velocity.

As the Schmidt number increases, the concentration decreases. This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity. Reductions in the velocity and concentration distributions are accompanied by simultaneous reductions in the velocity and concentration boundary layers.

An increase in chemical reaction parameter Kr leads to a decrease in both the values of velocity and concentration.

Appendix:

$$h = \frac{Pr}{1 + R} = m_1$$

$$m_2 = \frac{1}{2} \left( Sc + \sqrt{Sc^2 + 4ScK_r^2} \right)$$

$$m_3 = \frac{1}{2} \left( 1 + \sqrt{1 + 4M} \right)$$

$$m_4 = m_1 + m_2$$

$$m_5 = m_2 + m_3$$

$$m_6 = m_1 + m_3$$

$$m_7 = \frac{1}{2} \left( h + \sqrt{h^2 + 4hn} \right)$$

$$m_8 = \frac{1}{2} \left( Sc + \sqrt{Sc^2 + 4Sc(n + K_r^2)} \right)$$

$$m_9 = \frac{1}{2} \left( 1 + \sqrt{1 + 4(n + M)} \right)$$

$$m_{10} = m_3 + m_9$$

$$m_{11} = m_3 + m_7$$

$$m_{12} = m_3 + m_8$$

$$m_{13} = m_1 + m_9$$

$$L_1 = \frac{Gr}{(m_1^2 - m_1 - M)}$$

$$L_2 = \frac{Gc}{(m_2^2 - m_2 - M)}$$

$$L_3 = 1 + L_1 + L_2$$

$$L_4 = m_1^2 L_1^2$$

$$L_5 = m_2^2 L_2^2$$

$$L_6 = m_3^2 L_3^2$$

$$L_7 = m_1 m_2 L_1 L_2$$

$$L_8 = m_2 m_3 L_2 L_3$$

$$L_9 = m_1 m_3 L_1 L_3$$

$$L_{10} = \frac{L_4}{2m_1^2}$$

$$L_{11} = \frac{L_5}{4m_2^2 - 2m_1 m_2}$$

$$L_{12} = \frac{L_6}{4m_3^2 - 2m_1 m_3}$$

$$L_{13} = \frac{L_7}{m_4^2 - m_1 m_4}$$

$$L_{14} = \frac{L_8}{m_5^2 - m_1 m_5}$$

$$L_{15} = \frac{L_9}{m_6^2 - m_1 m_6}$$

$$L_{16} = L_{10} + L_{11} + L_{12} + L_{13} - L_{14} - L_{15}$$

$$L_{17} = \frac{L_{16}}{(m_1^2 - m_1 - M)}$$

$$L_{18} = \frac{L_{10}}{(4m_1^2 - 2m_1 - M)}$$

$$L_{19} = \frac{L_{11}}{(4m_2^2 - 2m_2 - M)}$$

$$L_{20} = \frac{L_{12}}{(4m_3^2 - 2m_3 - M)}$$

$$L_{21} = \frac{L_{13}}{(m_4^2 - m_4 - M)}$$

$$L_{22} = \frac{L_{14}}{(m_5^2 - m_5 - M)}$$

$$L_{23} = \frac{L_{15}}{(m_6^2 - m_6 - M)}$$

$$L_{25} = L_{17} - L_{18} - L_{19} - L_{20} - L_{21} + L_{22} + L_{23}$$

$$L_{26} = \frac{hA}{n}$$

$$L_{27} = \frac{AScm_2}{(m_2^2 - Scm_2 - Sc(n + K_r^2))}$$

$$L_{28} = 1 - L_{27}$$

$$L_{29} = \frac{Am_3L_3}{(m_3^2 - m_3 - (n + M))}$$

$$L_{30} = \frac{(Am_1L_1 - GrL_{26})}{(m_1^2 - m_1 - (n + M))}$$

$$L_{31} = \frac{(Am_2L_2 + GcL_{27})}{(m_2^2 - m_2 - (n + M))}$$

$$L_{32} = \frac{GrL_{26}}{(m_7^2 - m_7 - (n + M))}$$

$$L_{33} = \frac{GcL_{28}}{(m_8^2 - m_8 - (n + M))}$$

$$L_{34} = -L_{29} + L_{30} + L_{31} + L_{32} + L_{33}$$

$$L_{35} = (m_3L_3 - m_2L_2)m_9L_{34}$$

$$L_{36} = (m_3L_3 - m_2L_2)m_3L_{29}$$

$$L_{37} = h(m_3L_3L_{31} + m_3L_1L_{29} - m_2L_2L_{30})$$

$$L_{38} = (m_3L_3 - m_2L_2)m_2L_{31}$$

$$L_{39} = (m_3L_3 - m_2L_2 - m_1L_1)m_7L_{32}$$

$$L_{40} = (m_3L_3 + m_3L_{29} - m_2L_2)m_8L_{33}$$

$$L_{41} = m_1L_1m_9L_{34}$$

$$L_{42} = m_1^2L_1L_{30}$$

$$L_{43} = m_1L_1m_2L_{31}$$

$$L_{44} = \frac{L_{16}}{n}$$

$$L_{45} = \frac{2L_{10}}{(2m_1 - n)}$$

$$L_{46} = \frac{2m_2L_{11}}{(4m_2^2 - 2m_1m_2 - hn)}$$

$$L_{47} = \frac{2m_3L_{12}}{(4m_3^2 - 2m_1m_3 - hn)}$$

$$L_{48} = \frac{m_4L_{13}}{(m_4^2 - m_1m_4 - hn)}$$

$$L_{49} = \frac{m_5L_{14}}{(m_5^2 - m_1m_5 - hn)}$$

$$L_{50} = \frac{m_6L_{15}}{(m_6^2 - m_1m_6 - hn)}$$

$$L_{51} = \frac{L_{35}}{(m_{10}^2 - m_1m_{10} - hn)}$$

$$L_{52} = \frac{L_{36}}{(4m_3^2 - 2m_1m_3 - hn)}$$

$$L_{53} = \frac{L_{37}}{(m_6^2 - m_1m_6 - hn)}$$

$$L_{54} = \frac{L_{37}}{(m_5^2 - m_1m_5 - hn)}$$

$$L_{55} = \frac{L_{39}}{(m_{11}^2 - m_1m_{11} - hn)}$$

$$L_{56} = \frac{L_{40}}{(m_{12}^2 - m_1 m_{12} - hn)}$$

$$L_{71} = \frac{m_2}{(m_2^2 - m_2 - (n + M))}$$

$$L_{57} = \frac{L_{41}}{(m_{13}^2 - m_1 m_{13} - hn)}$$

$$L_{72} = \frac{m_3}{(m_3^2 - m_3 - (n + M))}$$

$$L_{58} = \frac{L_{42}}{(2m_1^2 - hn)}$$

$$L_{73} = \frac{L_{62}}{(m_7^2 - m_7 - (n + M))}$$

$$L_{59} = \frac{L_{43}}{(m_4^2 - 2m_1 m_4 - hn)}$$

$$L_{74} = \frac{L_{44}}{(m_1^2 - m_1 - (n + M))}$$

$$L_{60} = L_{44} + L_{45} + L_{46} + L_{47} + L_{48} - L_{49} - L_{50}$$

$$L_{75} = \frac{L_{45}}{(4m_1^2 - 2m_1 - (n + M))}$$

$$L_{61} = L_{51} + L_{52} - L_{53} - L_{54} - L_{55} - L_{56} - L_{57} + L_{58} + L_{59}$$

$$L_{76} = \frac{L_{46}}{(4m_2^2 - 2m_2 - (n + M))}$$

$$L_{62} = (Am_1 L_{60} + 2L_{61})$$

$$L_{63} = \frac{m_3 L_{25}}{(m_3^2 - m_3 - (n + M))}$$

$$L_{77} = \frac{L_{47}}{(4m_3^2 - 2m_3 - (n + M))}$$

$$L_{64} = \frac{m_1 L_{17}}{(m_1^2 - m_1 - (n + M))}$$

$$L_{78} = \frac{L_{48}}{(m_4^2 - m_4 - (n + M))}$$

$$L_{65} = \frac{2m_1 L_{18}}{(4m_1^2 - 2m_1 - (n + M))}$$

$$L_{79} = \frac{L_{49}}{(m_5^2 - m_5 - (n + M))}$$

$$L_{66} = \frac{2m_2 L_{19}}{(4m_2^2 - 2m_2 - (n + M))}$$

$$L_{80} = \frac{L_{50}}{(m_6^2 - m_6 - (n + M))}$$

$$L_{67} = \frac{2m_3 L_{20}}{(4m_3^2 - 2m_3 - (n + M))}$$

$$L_{81} = \frac{L_{51}}{(m_{10}^2 - m_{10} - (n + M))}$$

$$L_{68} = \frac{m_4 L_{21}}{(m_4^2 - m_4 - (n + M))}$$

$$L_{82} = \frac{L_{52}}{(4m_3^2 - 2m_3 - (n + M))}$$

$$L_{69} = \frac{m_5 L_{22}}{(m_5^2 - m_5 - (n + M))}$$

$$L_{83} = \frac{L_{53}}{(m_6^2 - m_6 - (n + M))}$$

$$L_{70} = \frac{m_6 L_{23}}{(m_6^2 - m_6 - (n + M))}$$

$$L_{84} = \frac{L_{54}}{(m_5^2 - m_5 - (n + M))}$$

$$L_{85} = \frac{L_{55}}{(m_{11}^2 - m_{11} - (n + M))}$$

$$L_{86} = \frac{L_{56}}{(m_{12}^2 - m_{12} - (n + M))}$$

$$L_{87} = \frac{L_{57}}{(m_3^2 - m_3 - (n + M))}$$

$$L_{88} = \frac{L_{58}}{(4m_1^2 - 2m_1 - (n + M))}$$

$$L_{89} = \frac{L_{59}}{(m_4^2 - m_4 - (n + M))}$$

$$L_{90} = (-L_{63} + L_{64} - L_{65} - L_{66} - L_{67} - L_{68} + L_{69} + L_{70})$$

$$L_{91} = (L_{71} + L_{72})$$

$$L_{92} = L_{74} + L_{75} + L_{76} + L_{77} + L_{78} - L_{79} - L_{80}$$

$$L_{93} = L_{81} + L_{82} - L_{83} + L_{84} - L_{85} - L_{86} - L_{87} + L_{88} + L_{89}$$

$$L_{94} = GrhA(m_1L_{74} - L_{64})$$

$$L_{95} = Grh(AL_{63} - 2L_{87})$$

$$L_{96} = Grh(AL_{65} + Am_1L_{75} + 2L_{88})$$

$$L_{97} = GrhA(L_{66} + m_1L_{76})$$

$$L_{98} = Grh(AL_{67} + Am_1L_{77} + 2L_{82})$$

$$L_{99} = Grh(AL_{68} + Am_1L_{78} + 2L_{89})$$

$$L_{100} = Grh(-AL_{69} - Am_1L_{79} + 2L_{84})$$

$$L_{101} = Grh(-AL_{70} - Am_1L_{80} - 2L_{83})$$

$$L_{102} = GrL_{73}$$

$$L_{103} = AGcL_{24}L_{91} + Gr(Am_1L_{90} + L_{73} - 2hL_{93} - hAm_1L_{92})$$

$$L_{104} = 2GrhL_{81}$$

$$L_{105} = 2GrhL_{85}$$

$$L_{106} = 2GrhL_{86}$$

$$L_{107} = m_1L_1 + m_2L_2 - m_3L_3$$

$$L_{108} = -(m_3L_{25} + 2m_1L_{48} + 2m_2L_{49} + 2m_3L_{20} + m_4L_{21})$$

$$L_{109} = m_1L_{17} + m_5L_{22} + m_6L_{23}$$

$$L_{110} = -(m_9L_{34} + m_3L_{29})$$

$$L_{111} = m_1L_{30} + m_2L_{31} + m_7L_{32} + m_8L_{33}$$

$$L_{112} = -(m_1L_{94} + m_3L_{95} + 2m_1L_{96} + 2m_2L_{97} + 2m_3L_{98} + m_4L_{99} + m_5L_{100} + m_6L_{101} + m_5L_{103} + m_{10}L_{104} + m_{11}L_{105})$$

$$L_{113} = (m_7L_{102} + m_{12}L_{106})$$

$$L_{114} = -(m_1L_{16} + m_5L_{14} + m_6L_{15})$$

$$L_{115} = 2m_1L_{10} + 2m_2L_{11} + 2m_3L_{12} + m_4L_{13}$$

$$L_{116} = L_{26}(m_1 - m_7)$$

$$L_{117} = -m_7L_{62}$$

$$L_{118} = -(m_1L_{44} + 2m_1L_{45} + 2m_2L_{46} + 2m_3L_{47} + m_4L_{48})$$

$$L_{119} = (m_5L_{49} + m_6L_{50})$$

$$L_{120} = -(m_{10}L_{51} + 2m_3L_{52} + 2m_1L_{58} + m_4L_{59})$$

$$L_{121} = m_6L_{53} + m_5L_{54} + m_{11}L_{55} + m_{12}L_{56} + m_{13}L_{57}$$

Figures:

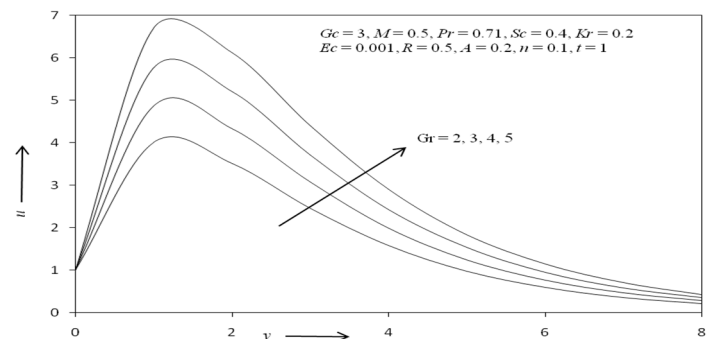


Fig.-(7.1): Effect of Gr on velocity profile



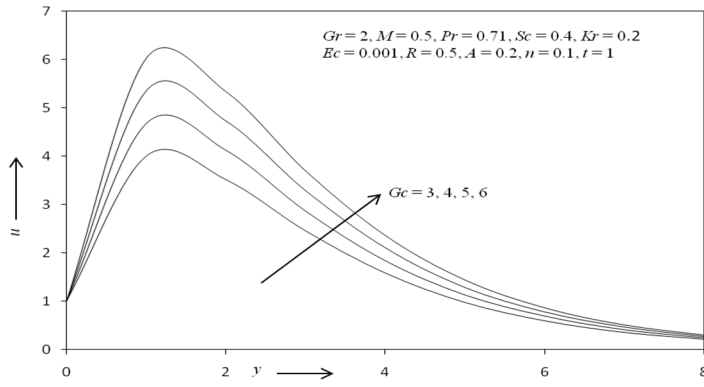


Fig.-(7.2): Effect of  $G_c$  on velocity profiles

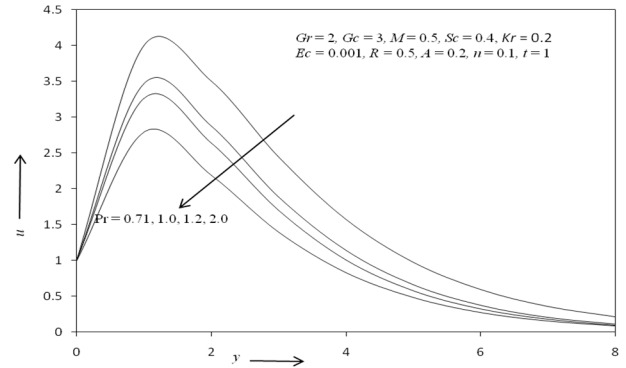


Fig.-(7.6): Effect of  $Pr$  on velocity profiles

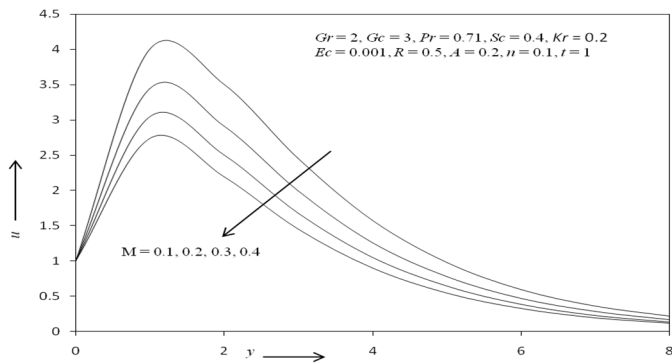


Fig.-(7.3): Effect of  $M$  on velocity profiles

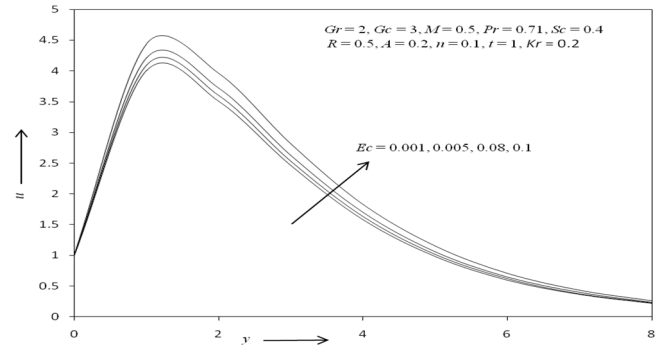


Fig.-(7.8): Effect of  $Ec$  on velocity profiles

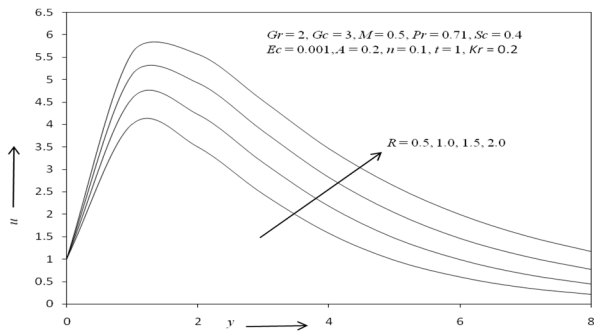


Fig.-(7.4): Effect of  $R$  on velocity profiles

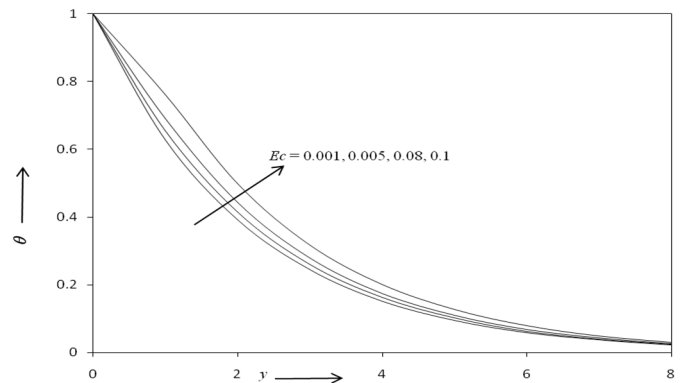


Fig.-(7.9): Effect of  $Ec$  on temperature profiles

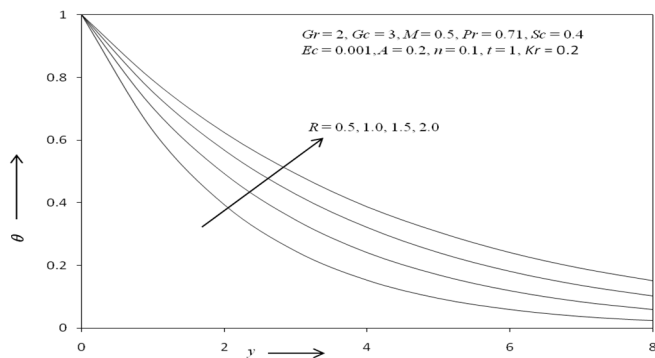


Fig.-(7.5): Effect of  $R$  on temperature profiles

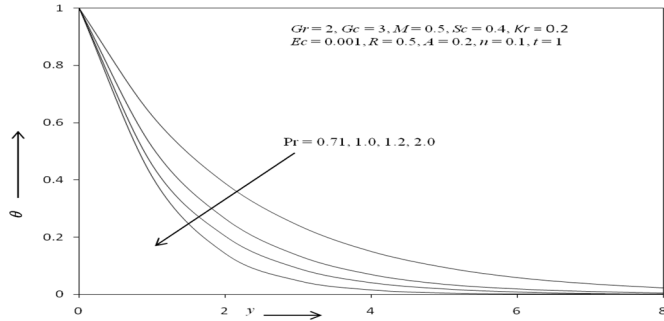


Fig.-(7.7): Effect of Pr on temperature profiles

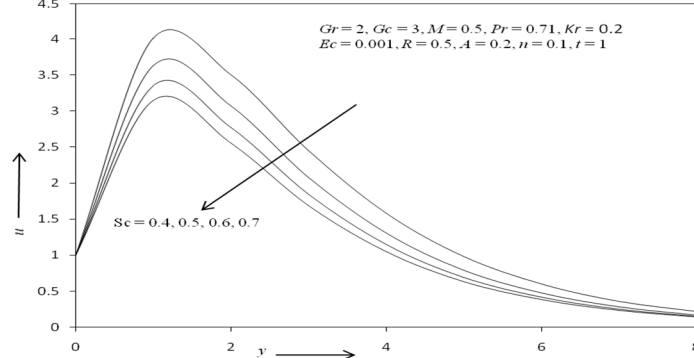


Fig.-(7.10): Effect of Sc on velocity profiles

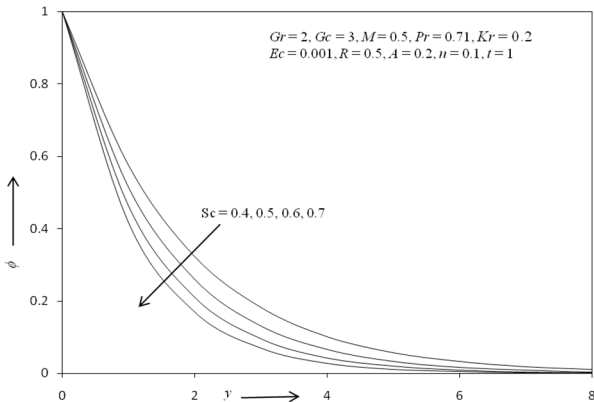


Fig.-(7.11): Effect of Sc on concentration profiles

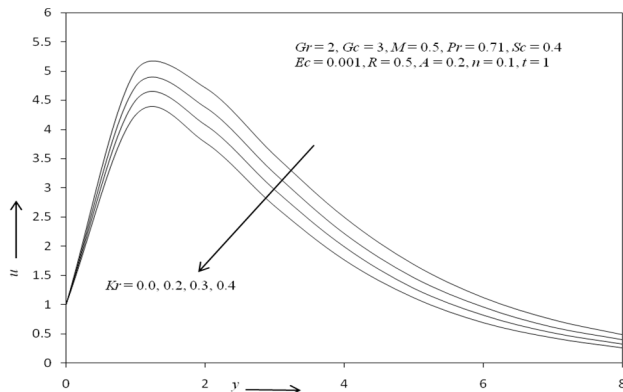


Fig.-(7.12): Effect of Kr on velocity profiles

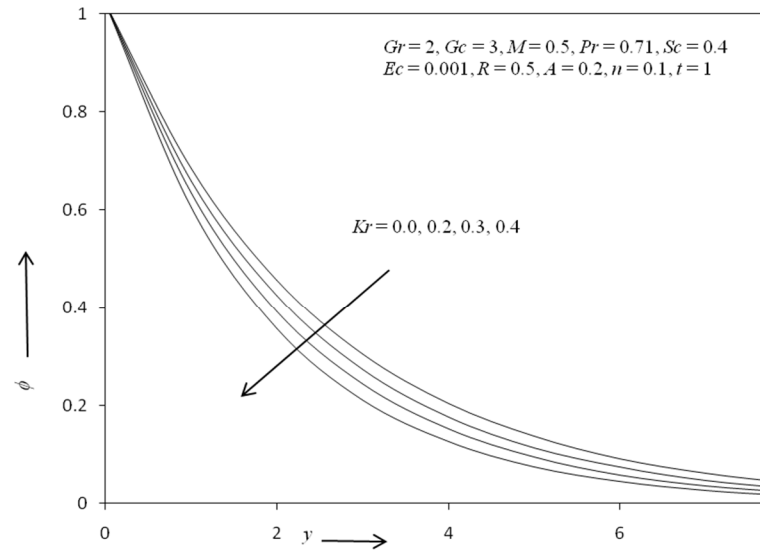


Fig.-(7.13): Effect of Kr on concentration profiles

## References

- Soundalgekar, V. M. and Takhar, H. S.: Modeling Measurement and Control, B51, pp. 31(1993).
- Hussain, M. A. and Takhar, H. S.: Heat and Mass Transfer, Vol. 31, pp. 234(1996).
- Rapits, A. and Perdakis, C.: Int. Journal Mech. And Engg., Vol. 8, pp. 125(1999).
- Chamka, A. J., Takhar, H. S. and Soundalgekar, V. M.: Chem. Engg. J., Vol. 84, pp. 335(2001).
- Muthucumaraswamy, R. and Ganesan, P.: Int. Journal Appl. Mech. And Engg., Vol. 44, pp. 104(2003).
- Prakash, J. and Ogulu, A.: Journal Pure and Applied Physics, Vol. 44, pp. 805(2006).
- Das, U. N. Deka, R. K. and Soundalgekar, V. M.: Forschung in Inge., Vol. 80, pp. 284(1994).
- Muthucumaraswamy, R.: Acta Mechanica, Vol. 155, pp. 65(2002).
- Brewster, M. Q.: Thermal Radiative transfer and Properties, John Willey & Sons, New York (1992).
- Reddy, M. G., Reddy, T. S. and Reddy, N. B.: Acta Ciencia Indica, Vol. XXXIV M, No. 2, pp. 639(2008).